

PRICING MODELS FOR HONG KONG WARRANTS

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## ABSTRACT

The present project compares the effectiveness of the application of warrant pricing models to Hong Kong data. Three types of models representing different approaches are chosen. These models are Black-Scholes option pricing model, the Shelton model, an econometric model, and the simplified Kassouf model which is a rule-of-thumb ad hoc model.

Fifteen warrants of Hang Seng Index constituent stocks are chosen. Data on warrant price, exercise price, maturity date, stock price, conversion ratio, dividend rate and the HIBOR rates spanning from 1987 to 1989 are collected. The first half of this period is used for estimating model parameters. The actual warrant prices of the second half of this period of data are compared with the predicted values from the three models. In order to find out the accuracy and prediction error of the models, the mean error, mean absolute error and root mean squared error are calculated. The effectiveness of the three models are then compared by performing the Wilcoxon Matched Pairs Rank Test.

The results show that the prediction errors of different models vary greatly across different warrants and no model dominates the others. This is obviously of value to investors who are considering subscribing to warrant stocks. To be concluded, both the effectiveness and the applicability of different models are unique to each warrant.



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## CHAPTER 1

### INTRODUCTION

#### Justification of the Research

A warrant is an option to buy a specified number of shares of a common stock at a specified price during a designated period. There are three main elements of a warrant; the exercise price which is the specified price at which a share can be bought, the exercise ratio which is the number of shares that could be purchased for every warrant and the expiration date.

#### Growing importance of warrant trading in Hong Kong

In recent years, warrant trading has assumed an increasing importance in the Hong Kong stock market. This can be witnessed by the increase in the number of warrants listed in Hong Kong and the growth in warrants trading volume. In January, 1987, the number of warrants listed in Hong Kong was 26 but it had grown to 78 by the end of that year.



The total market capitalization for securities and warrants for the Hong Kong market are shown below:

TABLE 1  
GROWTH OF WARRANT MARKET IN HONG KONG

<u>YEAR</u>	VALUE OF <u>EQUITIES</u> (HK\$ MIL.)	VALUE OF <u>WARRANTS</u> (HK\$ MIL.)	AS A % <u>OF EQUITIES</u>
1986	119,385.51	3,397.35	2.85
1987	352,627.86	18,269.87	5.18
1988	184,351.74	14,694.88	7.97

Source: Fact Book (1986-1988), published by Stock Exchange Of H.K. Ltd.

From the above, we can see that the relative importance of warrants in the securities market has grown significantly in recent years.

Also, plans are afoot to set up Warrant Funds in Hong Kong, showing the increasing recognition by funds manager on the potential profitability of warrant investments.

Why warrants are desirable investments?

The reasons for the increasing popularity of the warrant investment can be attributed to its risk and return



properties. Investors in warrants enjoy the benefits of leverage over investments in common stocks. Buying the warrants is like paying the first instalment or the deposit for the purchase of the common stock, while the exercise of the warrant right is similar to paying the remaining sum. When the price of a common stock fluctuates, the price of the warrant of that common stock fluctuates in the same direction with a magnified effect.

Since warrant trading has grown so fast in Hong Kong in recent years, a study of the application of different warrant models on the pricing of Hong Kong warrants is clearly of value.

### Research Objectives

This research focuses primarily on local warrants with emphasis on the valuation or pricing. The Black-Scholes Option Pricing Model, the Kassouf Warrant Pricing Model and the Shelton Warrant Pricing Model are used.

The main objectives are to :

- (1) compare the model pricing with the actual pricing to test the predictability of the models.
- (2) test and compare the effectiveness of the above mentioned pricing models on the local warrants.

It is important to note that the intention of our research is to find out and account for the predictability of the different models for local warrants valuation. Therefore, we will not try to challenge the underlying principles of the models being used.

We hope that the result of this research will shed some light on the warrant valuation characteristics of the local market.

## CHAPTER II

### METHODOLOGY

#### Data Source

The warrants of the Hang Seng Index constituent stocks are used while warrants of other local stocks are not included in this research. The main reasons are that the warrants of the Hang Seng Index constituent stocks are usually actively traded with large daily turnover which reduces the risk of the warrant price being manipulated or controlled by a small group of investors. Note that the warrants of Miramar Hotel expiring in 90, 92, 94 are not used in this research. Though Miramar is one of the Hang Seng Index constituent stocks, its warrants are not actively traded in the market and with very thin trading volume.

Weekly data for the period 1/1/87 to 31/12/89 are collected. The data include the warrant price, exercise price, stock price, expiration date, dividend yield, conversion ratio, and the HIBOR rate. The data are collected from the Hong Kong Economic Journal. While data for the number of shares issued and the number of warrants

outstanding come from the Securities Journal ( previously known as the Securities Bulletin). The table below summarizes the main features of the fifteen warrants chosen.<sup>1</sup>

TABLE 2

SUMMARY DATA OF 15 WARRANTS CHOSEN AS 31ST DEC., 1989.

	Conversion price	Conversion ratio	Expiry date
Great Eagle 92	1	1.000	Sep. 30
Great Eagle 94	2.8	1.000	Sep. 30
Hang Lung 92	9.5	1.000	Dec. 31
Hong Kong Electric 88	3.16	3.165	Dec. 31
Hong Kong Hotel 92	4.5	1.000	Dec. 15
Hong Kong Land 91	2.72	1.875	Dec. 31
Hopewell 91	1.81	1.105	Dec. 31
Hutchison Whampoa 89	11.85	0.844	Mar. 31
Jardine Matheson 92	16.49	0.606	Oct. 15
Kowloon Wharf 90	6.5	1.154	Dec. 31
Lai Sun 89	2.3	1.000	Dec. 31
New World 89	9	1.000	Dec. 31
New World 91	9	1.000	Dec. 31
Sun Hung Kai 90	7.55	2.212	Dec. 31
Sun Hung Kai 92	7.55	2.212	Dec. 31

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1. Abbreviation will be used for the warrant names in the following chapters. The full names are listed on Appendix 1.



## MODELS

Some published articles have discussed the pricing of Hong Kong warrants. One of the more recently published articles was 'The Effective Pricing Methods of Hong Kong Warrants' in the Hong Kong Economic Journal Monthly, May 1988.<sup>2</sup> The article introduced several models including the original Black-Scholes model, the Shelton model and the Guynemer model<sup>3</sup>. It also illustrated the models by using Hong Kong data. However, no articles were found researching the effectiveness of different warrant pricing models, especially applying the models to Hong Kong warrants.

A warrant is a security issued by the firm which promises to sell a specified number of shares to the holders for a fixed price (exercise price) at any time up to a stated date (maturity date). Therefore, a warrant is very much like an American call option written by the firm. The major difference is that exercise of warrants will increase the number of shares outstanding and thus dilutes the equity of the original stock holders.

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2. Original works in Chinese, 係學林, '估算認股證價格的有效方法', 信報月刊, 一九八八年五月.
  3. Guynemer model states that  $W = (S^2/4E) - (E/16)$  where W is the warrant price, S is the underlying stock price and E is the exercise price. According to Tsui (1988), this model tends to underestimate the warrant price and is therefore not commonly used.

As it is illegal to short sell both stocks and warrants in Hong Kong market, therefore, warrants cannot be used as a hedging tool like options in foreign markets. Warrants are mainly a speculative instrument in Hong Kong market due to its gearing effect and have the effect of margin trading of stock because the return of holding stock can be replicated by holding a portfolio of warrant and riskless bond.

Three models are chosen for evaluation in the project. For comparison, the models chosen should cover different approaches which are commonly used in valuing warrants. They also differ in complexity such that cost-effectiveness of the models can be compared. The models then chosen cover simple rule-of-thumb model, econometric and option pricing models.

#### Model 1 - Simplified Kassouf Model

Kassouf<sup>4</sup> defines the value of the warrant in relation to the stock as

$$Y = (A^2 + X^2)^{1/2} - A$$

where Y is the price of warrant, A is the exercise price, and X is the price of common stock. Kassouf thinks that

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4. Kassouf, S.T., Evaluation of Convertible Securities (Analytic Investors, Inc.: Maspeth, New York, 1962), p.26.



the owner of a warrant should require a leverage factor of about 2 as a fair compensation for the risks inherent in warrants. (Shelton, 1967) His equation implies that a warrant would appreciate 1.7 times the rate of the common stock if the common stock is selling at its option value; if the common stock is selling at half the option value, the warrant would appreciate twice as fast as the common stock.

The Kassouf model does not explicitly deal with factors like dividend yield foregone, whether the warrant is listed or not, or the longevity of the warrant. The dilution effect is clearly not dealt with in the model. The warrant price is simply related to the price of the stock, with any possible dilution from exercised warrants presumably already considered by investors. Any adjustment to the warrant price for dilution effect will mean double counting the effect. The model presented in due previous paragraph, is derived from Kassouf's original model. The original model is

$$W/E = [(S/E)^z + 1]^{1/z} - 1$$

where W denotes the price of the warrant,

E denotes the exercise price of the warrant,

S denotes the price of the stock,

and z is determined by a multiple regression process.

According to some empirical testing of Hong Kong warrants, the z value is quite close to 2. Therefore, for

simplicity,  $z = 2$  is chosen such that the equation becomes

$$W/E = [(S/E)^2 + 1]^{1/2} - 1$$

therefore,  $W = [S^2 + E^2]^{1/2} - E$

where the warrant price can easily be calculated.

## Model 2 - Shelton Model

The whole rationale of the Shelton model rests on the concept that warrant prices fall on a zone of plausible prices bounded with upper limits and lower limits determined by some relationship between the exercise price and the stock price.

The lower limit is the difference between stock price and exercise price, which is the minimum arbitrage value of the warrant. The upper limit is a new contribution to research on warrant pricing. The basic rationale is that when a stock price gets to be so high in relation to the warrant's option price, the warrant will sell for practically no more than its minimum exercise (or arbitrage) value regardless of the warrant's longevity. (Shelton, 1967) Holding a warrant will then require a higher rate of return because of the opportunity loss for the dividend yield foregone and the greater volatility of warrants, and the fact that in most cases warrants do not have the unlimited life of common stocks. Shelton claimed that the warrant would be sold at minimum arbitrage value



when the stock prices rose to four times the exercise price. If the stock price is less than four times the exercise price, Shelton argued that the highest plausible value of the warrant is  $3/4$  of the stock price.

The next question will be what should the position of warrant prices be within the plausible zone of values. Shelton argued that the position depended on how the warrant price responded to a series of factors which might affect warrant prices. There are six variables chosen: the longevity of the warrant, the dividend yield on the related stock, whether the warrant was listed (on the American Stock Exchange or traded over-the-counter), whether the warrant sold for more or less than prices which will not attract margin trading, the past volatility of the common stock, and the recent trend of the stock price. The relative importance of these variables are discovered by stepwise multiple regression. Some variables may be dropped if the regression analysis shows that they are insignificant.

The position of the warrant price will be determined by first dividing the time to maturity date by 72, and taking the fourth root of this value (Shelton, 1967), and then multiplying the value to the regression equation. For instance, Shelton, using the U.S. data, obtained the following result:  $[(M/72)^{1/4}](.47 - 4.25Y + .17L)$  where M is the time to maturity of the warrant, Y is the dividend

yield and  $L$  is whether the stock is listed or not. The resulting value will be a percentage. When multiplying to the width of the zone, which is the difference between the upper and lower limits of the plausible zone of values, and adding to the value of the lowest limit, the warrant price is determined.

According to the Shelton model (1967), warrant is priced by different equations under two different cases in this econometric model.

#### Case 1

If  $S \geq 4E$ , then

$$W = S - E$$

#### Case 2

If  $S < 4E$ , then

$$\begin{aligned} W &= (S - E) + [3S/4 - (S - E)] * (M/72)^{1/4} * (a - bY) \\ &= (S - E) + (E - S/4) * (M/72)^{1/4} * (a - bY) \end{aligned}$$

where  $M$  denotes number of months before expiration of the warrant,

$Y$  denotes dividend yield,

$a$  and  $b$  are constants to be determined by regression.

In order to apply linear regression process, the equation is transformed into

$$W = Z_1 + a*Z_2 - b*Z_3$$

where  $Z_1 = S - E$

$$Z_2 = (E - S/4) * (M/72)^{1/4}$$

$$Z_3 = (E - S/4) * (M/72)^{1/4} * Y$$

Then weekly closing values of E, S, M and Y from 1 January 1987 to 30 June 1988 are regressed against W to determine the value of a and b of each warrant. In order to obtain the predicted warrant price, the values of E, S, M and Y from 1 July 1988 to 31 December 1989 are inserted into the regressed equation

$$W = a_0 + a_1*Z_1 + a_2*Z_2 + a_3*Z_3$$

where  $a_0$ ,  $a_1$ ,  $a_2$  and  $a_3$  are regression coefficients.

For warrants which expired before 31 December 1989, the first portion of data is used for coefficients estimation and the last trading year data is used for prediction.

For warrants which did not start trading at 1 January 1987, the data up to 31 December 1988 is used for coefficients estimation while 1989 data is used for prediction.

### Model 3 - Black-Scholes Model

Black-Scholes model is an exact option pricing model based on the following assumptions :

1. Only European options are considered, that is, options that can be exercised only at maturity.
2. No transaction costs exist. Options and stocks



are infinitely divisible, and information is available to all without cost.

3. No imperfections exist in writing an option or selling a stock short.
4. The short-term interest rate is known and constant throughout the duration of the option contract. Market participants can both borrow and lend at this rate.
5. The stock pays no dividend.
6. Stock prices behave in a manner consistent with a random walk in continuous time.
7. The probability distribution of stock returns over an instant of time is normal.
8. The variance of the return is constant over the life of the option contract and is known to market participants.

The original Black-Scholes model is for European non-dividend paying options. Therefore, in order that it is used for warrant valuation, it is modified to account for the dilution effect, dividend payments and possibility of early exercise. The Black-Scholes formula for warrant is

$$W = [1/(1 + q)] * \{SN(x) - Er^t N[x - vt^{1/2}]\}$$

where  $q$  denotes the number of total newly issued shares upon exercise of all warrant/number of existing shares i.e. the dilution factor,

$r$  denotes the  $1 +$  risk free interest rate,

$t$  denotes the time to expiration in years,



$v$  denotes the standard deviation of  
returns of the underlying stock,

$$x = [\ln(S/Er^{-1})/vt^{1/2}] + (vt^{1/2})/2,$$

$N()$  denotes standard normal distribution function

Furthermore, the model is modified as follows:

1. The warrant value is calculated from the Black-Scholes formula after reducing the stock price by the present value of all the dividend payments on the underlying stock. That is, it is assumed the option will not be exercised early.
2. The warrant value is recalculated as in 1 but assuming that the warrant is exercised just before each ex-dividend date.

The maximum calculated warrant value of different ex-dividend date or expiration date is then the predicted price of the warrant.

To apply this model, the actual declared amounts of dividend and ex-dividend date are used if available. Otherwise, future dividend amounts and ex-dividend date are estimated from projection of past years data and the earning forecast from Hong Kong Economic Journal. Then, one and a half year or the first portion of actual weekly data is used for estimation of the volatility of the stock return as follows: Stock return of period  $j$ ,  $(R_j)$ , is defined as  $S_j/S_{j-1}$  where  $S_j$  and  $S_{j-1}$  are the stock prices

adjusted for dividend payment, stock split and bonus issues. Then  $\ln(R_j)$  is calculated. The square root of annualized variance of  $\ln(R_j)$  is used as the estimator of volatility ( $v$ ) of the stock return.

For the risk free interest rate, the Hong Kong Inter-bank Offer Rate (HIBOR) rate is taken as the proxy. One week HIBOR is used for one week expiration time; one month HIBOR is used for one week to one month expiration time; three months HIBOR is used for one to three months expiration time; six months HIBOR is used for more than three months expiration time.

### Testing Methods

#### Objectives

The objective of the test is to determine if, and how often the Black-Scholes model, Shelton model and Kassouf model predict warrant prices significantly closer to actual market prices. This can be done by examining the prediction errors of each model on the fifteen warrants chosen.

In order to find out which model is more effective, the three models are then compared against each other by performing the Wilcoxon matched pairs rank test.

# Test of accuracy

The purpose is to identify how and how much the model predicted prices deviate from the actual warrant price. In order to make the models comparable across fifteen warrants, the percentage deviation ( $D_i$ ) of predicted warrant price ( $W_m$ ) from the corresponding actual warrant price ( $W_a$ ) is calculated for each warrant for each model and divided by the model predicted warrant price. The formula is as follows:

$$\frac{W_a - W_m}{W_m}$$

Negative  $D_i$  means that the model has overestimated actual warrant price while positive  $D_i$  means that the model has underestimated actual warrant price.

The accuracy of each model is tested by the mean error (M.E.), mean absolute error (M.A.E.) and root mean squared error (R.M.S.E.) of the percentage deviation of model warrant price from the actual warrant price with definitions as follows:

$$M.E. = 1/n \sum D_i$$

Average of Percentage Deviation

$$M.A.E = 1/n \sum | D_i |$$

Average of Absolute Percentage Deviation



$$\text{R.M.S.E.} = [\sum (D_i)^2 / n]^{1/2}$$

Squared root of average squared percentage deviation

The mean error will show the dominating trend of either underestimation or overestimation of that particular warrant by that particular model. A small mean error does not necessarily mean smaller deviation of predicted warrant price from actual warrant price. It could be due to the fact that the negative and positive values of  $D_i$  have a cancelling effect. A small mean error may be the result of a cancelling out effect of a large negative and large positive  $D_i$ s. Therefore, the mean absolute error, with the signs removed, is calculated to give a picture of the magnitude of deviation of the model from the actual warrant prices. In this way, the mean absolute error serves as a measure of accuracy.

The root mean squared error is calculated to eliminate the cancelling out effect of negative and positive percentage deviations. Although the values of root mean squared error differ from the mean absolute error, a comparatively large mean error usually follows with a comparatively large root mean squared error.

The predictability of the models can also be shown by the biases of the mean errors. The number of cases with negative or positive mean errors roughly indicate how often

each model overestimates or underestimates that particular warrant.

In conclusion, the mean error shows the direction of deviation. Using the mean absolute error as a complementary reference, say, a small mean error and a small mean absolute error means that the predicted value is close to actual value. The findings can also be cross-checked with the root mean squared error. The biases will further show how frequent the model overestimate or underestimate the prices of that particular warrant.

### Rank Test

The three models are compared against each other for their effectiveness in predicting the warrant price. The effectiveness of the model can be indicated by how much each model warrant price deviates from the actual warrant price. A greater percentage deviation means that the model is less accurate in predicting that particular warrant. Therefore, the differences of percentage deviations of each pair of models are calculated. The Wilcoxon matched pairs rank test is then performed to find out how the positive and negative differences distribute. If the predictability of two models is not significantly different, the positive and negative rank values will cancel out and give a mean not significantly different from zero. The z-value of the

distribution is then tested against the hypothesis that the mean ( $u$ ) is zero.

If the hypothesis that  $u$  is zero is rejected, the sign of the  $z$ -value will show which model is better (which is the one with smaller percentage deviation). The significance level is taken as 5% in this rank test.

In summary, the steps for performing the rank test are as follows:

1. the differences of percentage deviations,  $D_i$ , of each pair of models are calculated;
2. the absolute value of the differences are ranked and the corresponding signs are then added to each rank;
3. the ranks are tested against the hypothesis that the mean of the rank distribution ( $X$ ) is zero, and the  $z$ -value, which is defined as  $z = (X - 0) / (d/n)$ , of each rank distribution is calculated, where  $d$  is the standard deviation and  $n$  is the number of observations;
4. the corresponding probability of each  $z$ -value are found out to see if the hypothesis should be accepted or



rejected;

5. if the hypothesis is rejected, the signs of the  $z$ -values are examined to determine which model is better.

### CHAPTER III

#### RESULTS & FINDINGS

##### Estimating the Shelton Model

Before discussing the accuracy levels and possible biases of the three competing models in any details, the coefficients for the Shelton model, for each warrant in Hong Kong chosen, have to be computed. In addition to due coefficients, measures of goodness-of-fit (or explanatory power) for the 15 estimated equations can be determined by the coefficient of determination,  $R^2$ .

##### Estimation of the Shelton Model

The coefficient of determination,  $R^2$ s, measures how the independent variables in use can explain the variation of the dependent variables. Therefore,  $R^2$  can be a rough indicator of goodness-of-fit of the model to predict the warrant prices. Table 3 presents the  $R^2$  and estimated coefficients for each of the 15 warrants estimated for the Shelton model.

TABLE 3

## REGRESSION EQUATIONS UNDER SHELTON MODEL

GE92:	Y	=	0.89	+	0.67Z <sub>1</sub>	-	0.71Z <sub>2</sub>	+	0.94Z <sub>3</sub>	
	(0.16)				(0.48)*		(1.95)		(6.82)	
	R <sup>2</sup>	=	0.96							
GE94:	Y	=	-6.12	+	1.32Z <sub>1</sub>	+	3.39Z <sub>2</sub>	+	4.56Z <sub>3</sub>	
	(0.10)				(0.11)*		(0.37)		(1.22)*	
	R <sup>2</sup>	=	0.96							
HL:	Y	=	0.66	-	0.02Z <sub>1</sub>	-	0.001Z <sub>2</sub>	+	0.004Z <sub>3</sub>	
	(0.13)				(0.04)		(0.06)		(0.003)	
	R <sup>2</sup>	=	0.03							
HKE:	Y	=	0.086	+	1.06Z <sub>1</sub>	-	2.82Z <sub>2</sub>	-	150.40Z <sub>3</sub>	
	(0.86)				(0.08)*		(2.76)		(50.33)*	
	R <sup>2</sup>	=	0.83							
HKH:	Y	=	-2.73	+	0.68Z <sub>1</sub>	+	1.39Z <sub>2</sub>	+	0.05Z <sub>3</sub>	
	(0.19)				(0.29)*		(0.96)		(0.03)	
	R <sup>2</sup>	=	0.45							
HKL:	Y	=	-4.23	+	1.21Z <sub>1</sub>	+	2.36Z <sub>2</sub>	-	8.82Z <sub>3</sub>	
	(0.30)				(0.07)*		(0.15)		(3.97)	
	R <sup>2</sup>	=	0.84							
HOPEWELL:	Y	=	0.52	+	0.81Z <sub>1</sub>	+	0.29Z <sub>2</sub>	+	3.81Z <sub>3</sub>	
	(0.11)				(0.06)*		(0.22)		(0.87)	
	R <sup>2</sup>	=	0.97							
HW:	Y	=	0.31	+	1.02Z <sub>1</sub>	+	1.61Z <sub>2</sub>	+	24.88Z <sub>3</sub>	
	(2.26)				(0.03)*		(2.11)		(88.99)	
	R <sup>2</sup>	=	0.98							
JM:	Y	=	-1.50	+	0.54Z <sub>1</sub>	+	0.72Z <sub>2</sub>	+	4.64Z <sub>3</sub>	
	(0.67)				(0.03)*		(0.04)		(1.05)	
	R <sup>2</sup>	=	0.95							
KW:	Y	=	-8.36	+	1.18Z <sub>1</sub>	+	2.30Z <sub>2</sub>	-	1.66Z <sub>3</sub>	
	(0.21)				(0.043)*		(0.12)		(1.51)	
	R <sup>2</sup>	=	0.976							
LS:	Y	=	-0.23	+	0.55Z <sub>1</sub>	+	0.77Z <sub>2</sub>	+	0.06Z <sub>3</sub>	
	(0.10)				(0.09)*		(0.14)		(0.80)	
	R <sup>2</sup>	=	0.53							
NW89:	Y	=	-3.50	+	0.87Z <sub>1</sub>	+	1.13Z <sub>2</sub>	-	0.43Z <sub>3</sub>	
	(0.41)				(0.054)*		(0.166)		(1.598)	
	R <sub>2</sub>	=	0.94							

$$\begin{array}{lcl} \text{NW91: } Y & = & -11.64 + 1.18Z_1 + 2.25Z_2 - 2.82Z_3 \\ (0.45) & & (0.11)^* (0.35)^2 (1.88)^3 \\ R^2 & = & 0.93 \end{array}$$

$$\begin{array}{lcl} \text{SHK90: } Y & = & -1.61 + 1.01Z_1 + 0.24Z_2 - 10.86Z_3 \\ (0.83) & & (0.07)^* (0.14)^2 (4.80)^3 \\ R^2 & = & 0.93 \end{array}$$

$$\begin{array}{lcl} \text{SHK92: } Y & = & -1.69 + 0.97Z_1 + 0.43Z_2 - 5.67Z_3 \\ (0.52) & & (0.04)^* (0.07)^2 (2.45)^3 \\ R^2 & = & 0.97 \end{array}$$

Notes:

Y = predicted warrant price

Z<sub>1</sub>, Z<sub>2</sub>, Z<sub>3</sub> = as defined in Shelton model

() = standard error of the parameters

R<sup>2</sup> = coefficient of determination

\* coefficients significant at 5% significance level

The results show that the R<sup>2</sup>s of most of the regression equations are large, with R<sup>2</sup> larger than 90%. For Hong Kong Electricity and Hong Kong Land greater than 80%. The only exceptions are warrants of Hang Lung, Lai Sun and Hong Kong Hotel. It means that the variables in the model, i.e. exercise price, stock price, time to expiration and the dividend yield of the stock can largely explain the variation in most of the actual warrant price.

A further examination reveals some plausible reasons for the small R<sup>2</sup>s for Hong Kong Electricity, Hong Kong Hotel and Hang Lung. For Hong Kong Hotel, there are two unfriendly acquisitions on shares and warrants of Hong Kong Hotels in 1987 and 1988. The major shareholder, Kadoorie



family, had to buy back lots of shares and warrants from the third party. The fact that lots of shares are in hands of minority in this buy back showed that prior to the unfriendly acquisitions, there were planned collections on the shares and warrants of Hong Kong Hotel. Therefore, the warrant prices may be speculative. The Shelton model fails to account for this speculation factor, and therefore results in a small  $R^2$ .

In 1987, Lai Sun International was formed from restructuring of Lai Sun Garment and no dividends was declared in the first few months after the formation. Therefore, the dividend yield input into the model was zero. However, the market might price the warrant based on future yield. So, the effect of the dividend is not fully accounted for in the model.

For Hang Lung case, there is no plausible factor observed to explain the small  $R^2$ .

The estimators are quite efficient in prediction although they are not necessarily unbiased (as shown by the large value of standard errors in some cases like Great Eagle 92 and Hutchison Whampoa.)

The signs for coefficients of  $Z_1$  are quite consistent (positively related to warrant price) and the coefficients for  $Z_1$  are statistically significant at 5% significance

level except for Hang Lung. For  $Z_2$ , the relationship with the warrant price is positive for most warrants except Hong Kong Electricity and Great Eagle 92 and Hang Lung. Most of the coefficients are statistically significant at 5% significance level. However, the relationship between  $Z_3$  and warrant price is quite obscure, with positive and negative signs. A careful look at the values and standard errors of the coefficients of  $Z_3$ s show that  $Z_3$  is not an important variable in explaining the variation of warrant prices. For eight warrants, the coefficients of  $Z_3$  are not statistically significant at 5% significance level. Within three standard errors for most of the coefficients of  $Z_3$ , the values of coefficients of  $Z_3$  approach zero. Referring to the regression model,  $Z_3$  is basically the multiple of  $Z_2$  by the dividend yield. And the dividend yield for stocks under analysis is not greater than 10% and hence making the  $Z_3$  values small. It can be interpreted as that the dividend yield, is not an important factor in determining Hong Kong warrant price.

There is no general conclusion on the values of coefficients for  $Z_1$ ,  $Z_2$  and  $Z_3$ . It seems that different warrant has different coefficient values.

### The Validity of the Models

Having computed the Shelton Model, a range of



predicted prices of the Shelton, Black-Scholes and Kassouf models can be formulated and direct comparisons of the models can be made.

It is believed that models with the mean absolute error greater than 20% for the percentage deviation of model warrant price from the actual warrant price should be rejected for predicting the corresponding warrant because there may be variables not covered in the model to predict the behaviour of the warrant. The missing variables may be, say, the speculation factor and market irregularities which are not formulated in the model. The results are shown in table 4.

As a result, the Black-Scholes model is not suitable for five warrants, which are Hong Kong Hotel, Hong Kong Land, Hopewell, Kowloon Wharf and Lai Sun.

The Shelton model is not suitable for eight warrants, which are New World 89, New World 91, Great Eagle 94, Hong Kong Hotel, Hong Kong Land, Hang Lung, Jardine Matheson and Kowloon Wharf.

The Kassouf model is not suitable for six warrants, which are Sun Hung Kai 90, New World 89, Hang Lung, Jardine Matheson, Kowloon Wharf and Lai Sun.

The efficiency of model prediction can be shown by the



range, mean and standard deviation of the M.A.E. of the three models. Excluding the inapplicable warrants, for Black-Scholes model, its mean M.A.E. is 9.11% with a standard deviation of 5.12% across ten warrants, and the M.A.E. ranges from 1.8% to 16.95% . For Shelton model, the mean of M.A.E. is 10.92% with a standard deviation of 4.96% and ranges from 3.47% to 20.34% across seven warrants. For Kassouf model, the mean of M.A.E. is 11.68% with a standard deviation of 5.56% and ranges from 4.8% to 19.45% across nine warrants. From this perspective, it can be seen that Black-Scholes model is comparatively more accurate in predicting the Hong Kong warrant prices.

TABLE 4

A SUMMARY OF MEAN ERROR, MEAN ABSOLUTE ERROR, AND ROOT MEAN SQUARED ERROR OF THE THREE MODELS  
PREDICTION

Black-Scholes Model

(in %)	<u>Mean Error</u>	<u>Mean Absolute Error</u>	<u>Root Mean Squared Error</u>
GE92	-9.45	9.45	11.13
GE94	13.62	15.03*	17.4
HL	-377.15	377.15*	564.34
HKE	-1.01	2.26*	3.11
HKH	-39.46	39.89*	46.29
HKL	-2.14	2.32*	2.77
HOPEWELL	-34.47	34.47*	36.13
HW	0.58	1.8	2.18
JM	16.77	16.95*	19.51
KW	-27.07	29.26*	43.36
LS	-58.95	70.14*	103.77
NW89	3.45	8.76	10.92
NW91	3.53	12.34	14.62
SHK90	3.3	10.42	13.11
SHK92	4.2	11.8	14.23
Average	-33.6166	42.802666667	60.191333333
Standard Deviation	94.11303	91.071153169	137.05664527

\* five warrants are inapplicable, with M.A.E. greater than 20%.

Shelton Model

(in %)	<u>Mean Error</u>	<u>Mean Absolute Error</u>	<u>Root Mean Squared Error</u>
GE92	-0.25	3.47*	5.34
GE94	-111.3	159.4*	331.9
HL	-22.23	30.13*	41.28
HKE	14.88	14.88*	15.76
HKH	-28.91	29.15*	30.61
HKL	-23.6	23.6*	27.92
HOPEWELL	-6.01	9.37	17.82
HW	8.37	8.37*	9.08
JM	-13.74	20.34*	23.71
KW	-451.23	480.7*	2038.63
LS	2.15	18.51*	32.22
NW89	67.9	182.67*	925.34
NW91	-59.44	59.64*	93.08
SHK90	5.95	9.06	11.32
SHK92	-0.62	10.93	14.33
Average	-41.2053	70.681333333	241.22266667
Standard Deviation	115.7915	121.69142341	533.1849884

\* eight warrants are inapplicable, with M.A.E. greater than 20%.

Kassouf Model

(in %)	<u>Mean Error</u>	<u>Mean Absolute Error</u>	<u>Root Mean Squared Error</u>
GE92	4.96	6.18	7.26
GE94	14.18	17.2	19.84
HL	33.81	33.81 *	35.54
HKE	4.41	4.8	5.29
HKH	13.39	13.39	14.93
HKL	9.37	9.37	9.73
HOPEWELL	9.56	10.03	11.36
HW	5.35	5.35 *	5.87
JM	23.49	23.49 *	24.87
KW	21.37	21.86 *	25.38
LS	54.84	54.84 *	57.33
NW89	43.41	43.41 *	45.1
NW91	19.3	19.36 *	21.18
SHK90	25.53	25.53 *	26.38
SHK92	18.68	19.45	21.43
Average	20.11	20.538	22.099333333
Standard Deviation	14.10126	13.8942329523	14.3292872429

\* six warrants are inapplicable, with the M.A.E. greater than 20%.

The bias of the prediction of warrant prices under each model can be shown by the signs of mean errors of the fifteen warrants (refer to table 5). It can be seen that the Black-Scholes model has overestimated (positive mean error) seven warrants and underestimated (negative mean error) eight warrants. The Shelton model has overestimated five warrants and underestimated ten warrants. The Kassouf model has consistently overestimated all the warrants. But according to the M.A.E., the Kassouf model appears more accurate.



TABLE 5

A SUMMARY OF SIGNS OF MEAN ERRORS OF VARIOUS WARRANTS UNDER THREE MODELS

<u>Black-Scholes</u>		<u>Shelton</u>		<u>Kassouf</u>
<u>+ve signs</u>	<u>-ve signs</u>	<u>+ve signs</u>	<u>-ve signs</u>	<u>+ve signs</u>
SHK92	GE92	SHK90	SHK92	SHK92
SHK90	HKE	NW89	NW91	SHK90
NW89	HKH	HKE	GE92	NW89
NW91	HL	HW	GE94	NW91
GE94	HOPEWELL	LS	HKH	GE92
HW	KW		HKL	GE94
JM	LS		HL	HKE
	HKL		HOPEWELL	HKH
			JM	HKL
			KW	HL
				HOPEWELL
				HW
				JM
				KW
				LS
<hr/>				
7	8	5	10	15
=	=	=	==	==

Overestimation or Underestimation

Warrants with negative mean errors imply that the models have underestimated the warrant prices, and vice versa.

Excluding the inapplicable warrants, which have an absolute mean error over 20% (as mentioned in previous section), Black-Scholes model underestimates three warrant prices, i.e. Great Eagle 92, Hong Kong Electric and Hong Kong Land while overestimate other seven warrants : New World 89 and 91, Sun Hung Kai 90 and 92, Great Eagle 94, Hopewell and Jardine Matheson. Shelton model

underestimates three warrant prices, i.e. Great Eagle 92, Hopewell and Sun Hung Kai 92 while overestimates four warrants prices of Sun Hung Kai 90, Hong Kong Electric and Hutchison Whampoa. Kassouf model has consistently overestimated all the nine warrant prices of New World 91, Sun Hung Kai 92, Great Eagle 92 and 94, Hong Kong Electric, Hong Kong Land, Hopewell and Hutchison Whampoa.

#### Mean Error vs Mean Absolute Error

If the mean absolute error is greater than the mean error in magnitude, it means that there is cancellation of positive and negative errors. That is, the model sometimes over-estimates the warrant prices while sometimes under-estimates them. Then, a model with a small mean error while a significantly larger M.A.E. gives less reliable predictions.

For the warrants with the mean error more or less equal to mean absolute error, the percentage deviations are always in one direction, i.e. either positive or negative. That is, the model consistently over- or under-estimates the prices of the warrant.

For certain warrants, the Black-Scholes model has consistently underestimated (with a negative M.E.) the actual warrant price, i.e. Great Eagle 92, Hong Kong Land,

Hang Lung, Hopewell and Kowloon Wharf; or overestimated (with a positive M.E.) the actual warrant price, i.e. Jardine Matheson and perhaps Great Eagle 94.

The Shelton model has consistently underestimated the actual warrant price for New World 91, Hong Kong Hotel, and Hong Kong Land; and overestimated the actual warrant price for Hong Kong Electricity and Hutchison Whampoa.

The Kassouf model has consistently overestimated the actual price for all the warrants which the model is applicable.

For other warrants, the models randomly over- or under-estimate their prices.

#### Ranking Of The Models

Having considered the accuracy of the different models in the previous section, we would like to compare the models in this section to see whether we can tell which one is better than the others.

The pricing of the models are compared by the Wilcoxon's matched pairs rank test. The percentage deviations of each model pricing are calculated by subtracting the model pricing by the actual pricing and then divided by the model pricing. Two models are compared



at each time. The differences between the absolute percentage deviations of the two models for all warrants under test are ranked by their absolute magnitudes in ascending order and were given the rank 1,2,3,4,....etc. Then the rankings were given the same sign as the differences. The mean and standard deviation were calculated for the rankings. We performed the Z-test on the hypothesis that the mean percentage deviations of the two models are equal

$$H_0 : U_s - U_k = 0$$

at a 5% level of significance.

The Z values and the corresponding P(r) tail area for the 15 warrants comparing the three models are summarised in table 6.

TABLE 6  
RESULTS OF RANKING TEST

	Rank (S-K)		Rank (BS-S)		Rank (BS-K)	
	<u>Z value</u>	<u>P(r)</u>	<u>Z value</u>	<u>P(r)</u>	<u>Z value</u>	<u>P(r)</u>
Great Eagle 92	-8.32	0	14.48	0	6.11	0
Great Eagle 94	-1.27	0.103	-10.03	0	-4.53	0
Hang Lung	-0.93	0.177	8.37	0	10.88	0
Hong Kong Electric	10.92	0	-11.19	0	-3.73	0
Hong Kong Hotel	7.53	0	3.65	0	7.31	0
Hong Kong Land	7.59	0	-14.08	0	-15.08	0
Hopewell	-7.93	0	12.39	0	14.87	0
Hutchison Whampoa	13.97	0	-13.69	0	-9.67	0
Jardine Matheson	-1.36	0.087	-9.98	0.165	-15.59	0
Kowloon Wharf	2.69	0.004	-11.74	0	2.02	0.022
Lai Sun	-9.85	0	-7.19	0	0.61	0.271
New World 89	-0.78	0.022	-9.89	0	-5.62	0
New World 91	4.25	0	-7.51	0	-5.51	0
Sun Hung Kai 90	-15.24	0	1.17	0.121	-11.33	0
Sun Hung Kai 92	-4.85	0	2.1	0.018	-5.01	0

If each tail area  $P(r)$  of the Z value is greater than 2.5%, we cannot reject the hypotheses that the mean percentage deviations of the two models are equal at a 5% level of significance. Otherwise, we would conclude that the mean percentage deviations of the two models are statistically different at a 95% level of confidence.

Which model is better then? Referring to table 6, take Great Eagle 92 as an example. Comparing the Shelton and the Kassouf models,  $P(r) < 2.5\%$  , which means that we

have more than 95% level of confidence that the mean percentage deviations of the two models are not equal. The Z value for rankings of Shelton's absolute percentage deviations less Kassouf's absolute percentage deviations is negative, which implies that the absolute percentage deviation of the Shelton model are on average less than those of the Kassouf. That is, Shelton is a better predictor of the actual price than Kassouf in this case. Similarly, Shelton is better than Black-Scholes and also Kassouf is better than Black-Scholes.

The rankings of the three models for the 15 warrants are summarized in table 7. The best model in predictive power is given a score of "1", the second best a score of "2" and the model with the lowest predictive power a score of "3". If we cannot tell statistically which of the two models is better, the score will be shared equally between them.



TABLE 7

RANKING OF THE MODELS

	SHELTON	KASSOUF	BLACK-SCHOLES
Great Eagle 92	1	2	3
Great Eagle 94	2	3	1
Hang Lung	1	2	3
Hong Kong Electric	3	2	1
Hong Kong Hotel	2	1	3
Hong Kong Land	3	2	1
Hopewell	1	2	3
Hutchison Whampoa	3	2	1
Jardine Matheson	2	3	1
Kowloon Wharf	2	1	3
Lai Sun	1	2.5	2.5
New World 89	2.5	2.5	1
New World 91	3	2	1
Sun Hung Kai 90	1.5	3	1.5
Sun Hung Kai 92	2	3	1
	-----	-----	-----
	30	33	27
	==	==	==

From table 7, Shelton scores 30 points, while Kassouf scores 33 points and Black-Scholes scores 27 points. By just looking at the scores of each model, it seems that Black-Scholes is the most powerful predictive model while Kassouf is the least powerful predictive one. However, from a statistical point of view, we cannot conclude which one is more predictive than the others.

Sensitivity Analysis

The following analysis tries to identify the effect of changes of the independent variables on the warrant prices for the three models. These independent variables include exercise price, price of the underlying stock, time to expiration, volatility of the underlying stock, risk free interest rate.

### Simplified Kassouf Model

According to the model, warrant price depends on only two independent variables : exercise price and price of the underlying stock.

The changes of the warrant price with respect to changes of exercise price is shown by :

$$\partial W / \partial E = E / [(S^2 + E^2)^{1/2}] - 1 < 0$$

This shows that when there is a change in the exercise price of the warrant, the warrant price will changes in the opposite direction. However, the magnitude of sensitivity depends on the existing exercise price and stock price. The greater the existing exercise price E, the greater the sensitivity while the greater the stock price S, the smaller the sensitivity of the warrant price.

The changes of the warrant price with respect to changes of stock price is shown by :

$$\partial W / \partial S = S / [(S^2 + E^2)^{1/2}] > 0$$

The value of  $S / (S^2 + E^2)^{1/2}$  is always greater than 0. Then the warrant price moves in same direction with the stock price. The magnitude of sensitivity of warrant price also depends on existing stock price and exercise

price. The greater the stock price, the greater the sensitivity while the greater the exercise price, the smaller the sensitivity.

### Shelton Model

According to the model, the warrant price depends on the exercise price, stock price, time to expiration and the dividend yield of the stock.

The changes of the warrant price with respect to exercise price is shown by :

$$\partial W / \partial E = - 1 + (a - bY) * (M/72)^{1/4}$$

Therefore, the sensitivity depends on the value of dividend yield of the stock and the time to expiration of the warrant. The greater the dividend yield, the smaller the sensitivity. While the longer the time to expiration, the greater the sensitivity of the warrant price. The direction of warrant price changes with respect to the changes of exercise price cannot be explicitly determined from the above equation. For short time to expiration warrant and not too large dividend yield rate, it can be expected that the sensitivity is a negative number. That is, the change in warrant price will be in an opposite direction with that of the change in exercise price. This



is very reasonable as the higher the exercise price, the less probable that the warrant will become in-the-money and therefore the less valuable is the warrant.

The changes of the warrant price with respect to stock price is shown by :

$$\partial W / \partial S = 1 - (1/4) * (a - bY) * (M/72)^{1/4}$$

Therefore, the sensitivity also depends on the value of dividend yield and the time to expiration of the warrant. But the relationship is opposite to that of exercise price sensitivity. That is, the greater the dividend yield, the greater the sensitivity and the longer the time to expiration, the smaller the warrant sensitivity. The sign of the sensitivity value is also cannot be explicitly determined. However, it can reasonably expected that it has a positive value, i.e. the greater the stock price, the greater the warrant price.

Mathematically, the relationship of the two sensitivities is :

$$\partial W / \partial S = 1 - (1/4) * \partial W / \partial E + 1)$$

The changes of the warrant price with respect to time to expiration is shown by :

$$\begin{aligned} \partial W / \partial M &= (1/4) * (1/72)^{1/4} * (E - S/4) * (a - bY) \\ &* (M^{3/4}) > 0 \end{aligned}$$

Then, the warrant price will rise if time to expiration is prolonged and vice versa. The magnitude of price sensitivity with respect to time to expiration depends on the value of stock price, exercise price, the dividend yield and the time to expiration. The greater the exercise price, the greater the sensitivity while the greater the stock price, the dividend yield and the time to expiration, the smaller the sensitivity.

Finally, the changes of warrant price with respect to the changes of dividend yield is shown by :

$$\partial W / \partial Y = -b * (E - S/4) * (M/72)^{1/4} < 0$$

This agrees with the common sense that the larger the dividend payment, the smaller the warrant price as warrant does not enjoy benefits of dividend payment. So, the changes in warrant price is opposite to that of the changes of the dividend yield. The magnitude of the sensitivity depends on the value of the exercise price, stock price and time to expiration. The greater the exercise price and time to expiration, the greater the sensitivity, while the greater the stock price, the smaller the sensitivity.

Out of the above sensitivities of Shelton model, sensitivity with respect to stock price is of most significant. This is because other variables can be forecasted with reasonably certainty, also the maturity

date and exercise price seldom changes. So by looking at with warrant sensitivity with respect to stock price, investor can estimate the impact of stock price on warrant price.

### Black-Scholes Model

According to the model the warrant price depends on the stock price, the exercise price, the time to expiration, the volatility of the stock and the interest rate. According to Cox and Rubinstein (1985), the warrant price sensitivities with respect to different independent variables are as follow :

$$\partial W / \partial S = 1 - D * N(x) > 0$$

$$\partial W / \partial E = D * -r^{-t} * N(x - \sqrt{vt}) < 0$$

$$\partial W / \partial t = D * (S * v) / (2 * t^{1/2}) * N'(x) + E * -r^{-t} * \ln(r) * N(x - \sqrt{vt}) > 0$$

$$\partial W / \partial v = D * S * t^{1/2} * N'(x) > 0$$

$$\partial W / \partial r = D * t * E * r^{-(t+1)} * N(x - \sqrt{vt}) > 0$$

where D is the warrant dilution factor.

The warrant price will move in the same direction of the changes in independent variables if the sensitivities are positive while it will move in the opposite direction for negative value sensitivities value. So, an increase in stock price, time to expiration, volatility and interest rate will increase warrant price. Out of all the



independent variable, only an increase in exercise price will decrease warrant price. The effect of a change in dividend payment can be considered has an opposite change in stock as stock price is adjusted by subtracting present value of dividend payment from the stock price.

Out of the above-mentioned different warrant price sensitivities, the ones with respect to volatility and stock price are of most significant. As the other independent variables can be observed from market directly, volatility is the one that has to be estimated from past data. It is also doubtful whether this value is consistent over time. From above,  $N'(x)$ ,  $r^{-(t+1)}$  and  $D$  is smaller than 1. So the magnitude of the sensitivity largely depends on stock price  $S$ . The higher the stock price value, the greater effect does an inaccurate estimation of volatility have on the warrant price and the effect is directly related.

#### Elasticity of warrant price

The sensitivity of warrant price with respect to different independent variables are stated in previous paragraph. However, sensitivity only shows the magnitude of the changes of warrant price. For comparison purpose, elasticity of warrant price which shows the percentage change of warrant price with respect to a percentage change

of a variable, is an useful indicator. The elasticity of warrant price can be directly calculated from its sensitivity by the following equation :

$$\begin{aligned} & \text{Elasticity of warrant price, } W \text{ with respect to } X \\ &= (\partial W/W) / (\partial X/X) \\ &= X/W * \text{Sensitivity of } W \text{ with respect to } X. \end{aligned}$$

#### Warrants issued by the same company

Some companies issue warrants with different expiration dates, for example, Sun Hung Kai issued warrants expiring on 31 December 1990 and 31 December 1992. This provides a good opportunity to check the differences between actual price from that of being predicted by the models.

According to the Kassouf model, warrants with the same exercise price and stock price but with different expiration date should have the same price.

For Shelton model, there should be a difference in warrant prices equalling to  $(E - S/4) * (a - bY) * [(M_1/72)^{1/4} - (M_2/72)^{1/4}]$  if the warrants issued by same company have same exercise price.

For Black-Scholes model, the difference in warrant price is quite complicated as the stock price has to be

adjusted by the present value of future dividend payments due to different expiration dates, but generally the longer the time to expiration, the more valuable is the warrant. So, the model predicts warrant with a longer time to expiration has a greater price than that of a shorter time to expiration.

As an illustration, we can make use of Sun Hung Kai 90 and 92 warrant to check the model predictions against the actual. From the graph (Appendix 2), the price of longer time to expiration is actually greater than that of shorter time to expiration (positive price difference). So, Kassouf model fails to account for the factor, time to expiration, because the market values the warrant with a consideration of this factor. The other two models both predict a positive price difference. But the Black-Scholes model predicts a price difference closer to the actual than that predicted by the Shelton model.



## CHAPTER IV

### CONCLUSION

From the above findings, we can draw several conclusions.

1. There is no distinct and conclusive differences in the predictive power of the three models after eliminating the inapplicable warrants. But on average, Black-Scholes model gives a more accurate prediction than Shelton model which in turn is more accurate than Kassouf model. This is not surprising as the number of variables that Black-Scholes model account for is greater than that of Shelton and Kassouf.
2. It seems that there is a trade-off between model complexity and accuracy. If cost-effectiveness is taken into account, simplified Kassouf model perform better when compared with more complex models like Shelton and Black-Scholes. This is especially true if a rough estimation of warrant price is required in a matter of minutes. However, the tendency that this model always

over-estimates warrant prices must be taken into account when being applied.

3. The applicability of the models varies greatly among different warrants. A model can be applicable to a warrant while at the same time not applicable to another warrant of the same company. So, the selection of models is quite unique for each warrant. However, this applicability of the model can be maintained for a period of time. If it is found that the model fits the warrant for the historical data, it is reasonably confident that the model will have good predictive power in the near future.

## CHAPTER V

### LIMITATION OF MODELS & FURTHER RESEARCH

The existence of prediction errors and non-applicability of the models for certain warrants can be attributed to a number of reasons. These include :

1. The models themselves do not take into account all the factors that actually affect the warrant prices. Certain warrant prices may be subject to a certain form of price manipulation which is possible because Hong Kong stock market may not be as efficient as larger markets in the U.S. or U.K. In addition, share holding distributions are very uneven.
2. Certain assumptions are not valid. For Shelton model, it is doubtful whether coefficients of the variables are consistent over time. The existence of non-zero constant term also indicates the insufficiency of the model. For Black-Scholes model, it is also doubtful whether the assumption that the volatility of the stock will be constant over time is valid.



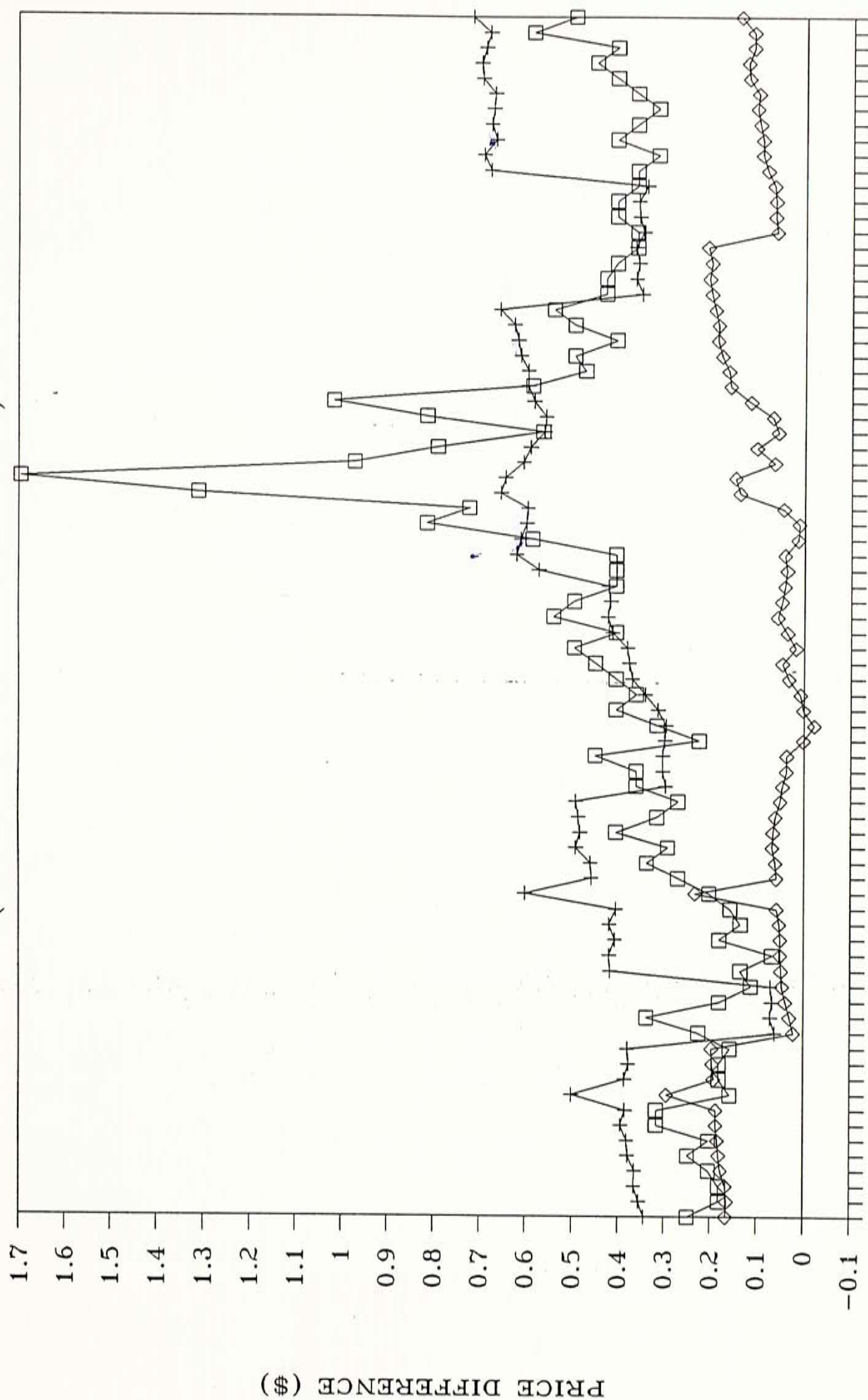
Subject to these limitations, we propose that further researches can be performed in the areas like how to improve the estimation of stock volatility and consistency of the value, the required time horizon for which revision of regression coefficients are required, and the extension of the Shelton model to take into account more relevant factors.

**Appendix 1: Abbreviation for warrant names**

Great Eagle 92	-	GE92
Great Eagle 94	-	GE94
Hang Lung 92	-	HL
Hong Kong Electric 88	-	HKE
Hong Kong Hotel 92	-	HKH
Hong Kong Land 91	-	HKL
Hopewell 91	-	HOPEWELL
Hutchison Whampoa 89	-	HW
Jardine Matheson 92	-	JM
Kowloon Wharf 90	-	KW
Lai Sun 89	-	LS
New World 89	-	NW89
New World 91	-	NW91
Sun Hung Kai 90	-	SHK90
Sun Hung Kai 92	-	SHK92

# SHK 90 AND 92 WARRANT PRICE DIFFERENCE

(1 JULY 1988 TO 31 DECEMBER 1989)





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